COPPER GRADE PATTERNS AT DIFFERENT SCALES : AN EMPIRICAL STUDY

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ILLUSTRATIONS

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ABSTRACT

A sulphite orebody was recognized at a metric scale for reserve evaluation purposes. A drilling campaign was performed and 1 m support samples were assayed for copper. In a later phase, some cuttings were splitted in 2.5 mm pieces in order to measure ore dressing control parameters. Each 2.5 mm sample was also assayed for copper. Variograms of the copper grade were calculated for the two sets of samples and a striking similarity was noticed between the general "attitude" of the variograms at different scales, suggesting a structural repetition.

A scale-free grade pattern is hypothesised, leading to a reduced variogram model in h^{α} which depends only on the sampling interval for each set of data.

RESUME

La régionalisation des teneurs en cuivre à des échelles différentes

Une étude empirique

Afin d'en évaluer les réserves minières, un gisement de sulfures a été reconnu par sondages à différentes échelles. Un premier ensemble de données constitué par des tronçons de sondages de 1 m a été utilisé pour la détermination des teneurs en cuivre. Dans une phase finale, quelques sondages destructifs ont été effectués et des analyses en Cu ont été faites sur des tronçons de sondages de 2.5 mm afin d'évaluer les paramètres de contrôle liés à la valorisation du minerais. Les variogrammes des teneurs en cuivre ont été calculés sur ces deux ensembles de données. Une ressemblance remarquable peut être observée entre ces deux courbes calculées pour ces deux échelles, ce qui suggère une répétition structurale.

Un modèle de régionalisation indépendante de l'échelle à laquelle est effectué l'échantillonnage est proposé, le variogramme utilisé est de la forme h^{α}.

A - EXPERIMENTAL EVIDENCE

In a sulphite orebody, two sets of samples were assayed for Cu :

- . the first set, denoted "MET" in the following, is composed by 616 core samples taken 1 m apart in drill-holes. The length of each core is also 1 m.
- . the second set, denoted "MIL", contains 1130 small samples (2.5 mm long) taken at a constant mesh of 2.5 mm in some cores, assumed to be representative of the ore type being studied.

Variograms down-hole were calculated in both sets (cf. table of experimental variogram values in Annex A). For the first set, the average reduced variogram is presented in figure 1 in ordinates, values of the variogram ($\gamma_{MET}(h)$) divided by the experimental variance (VAR) are plotted for multiples of the basic step size (1 m). The average is denoted by m and the number of points by N.

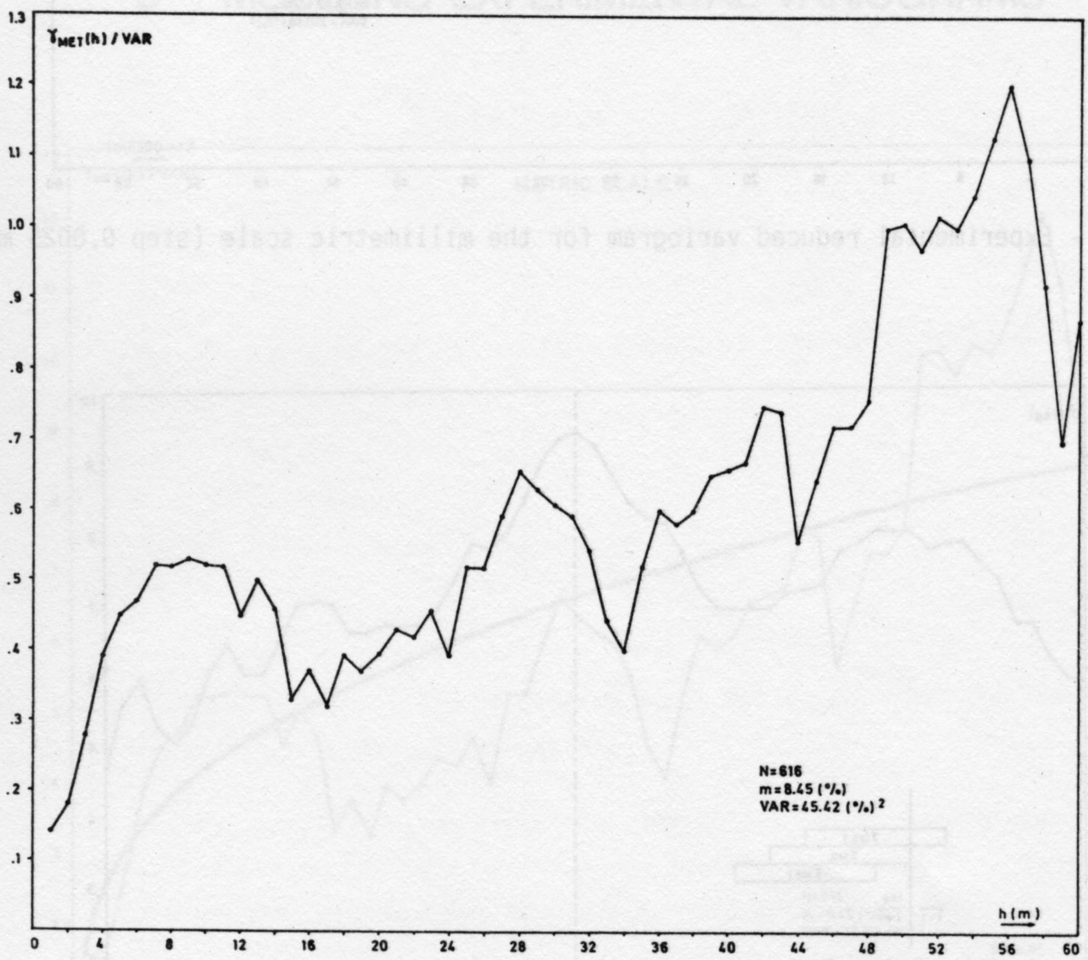

Fig. 1 - Experimental reduced variogram for the metric scale (step 1 m)

For the second set, the average reduced variogram is shown in figure 2. The step is now 2.5 mm for this set of data.

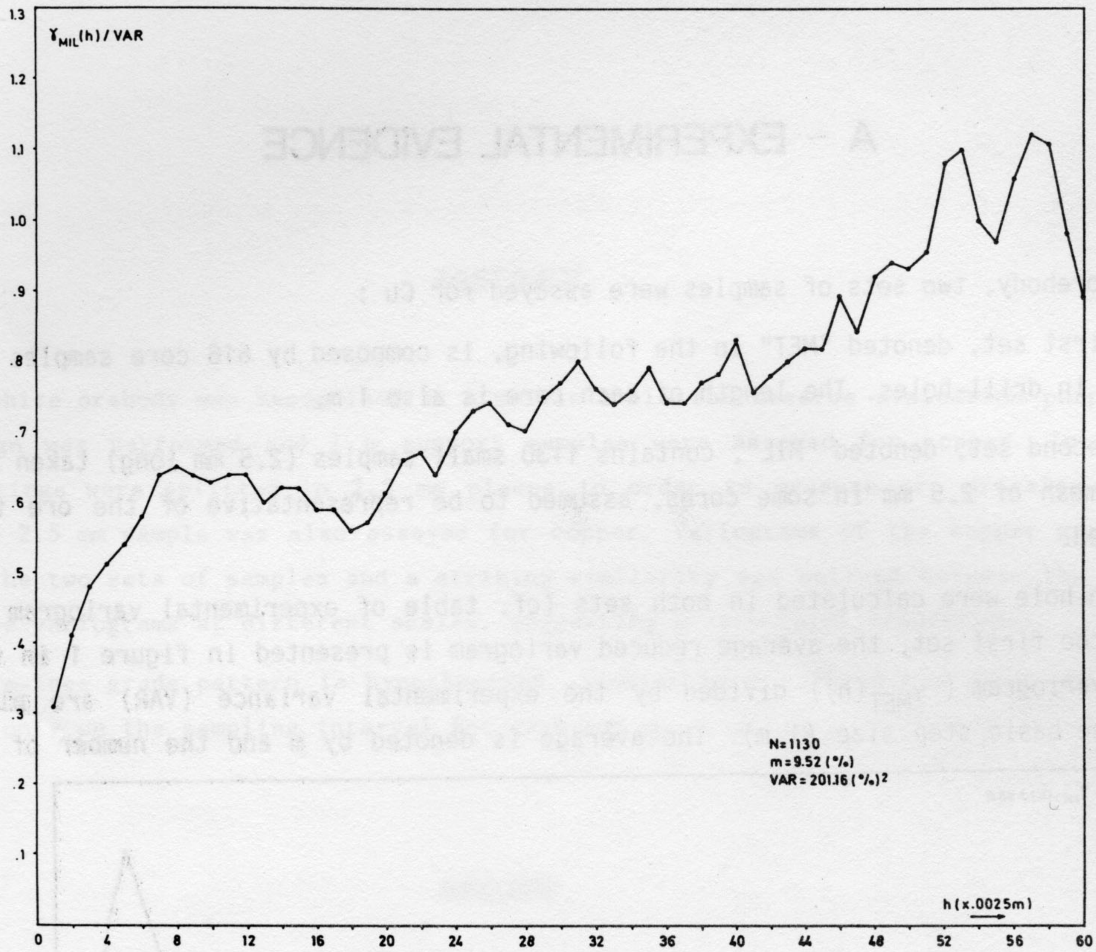

Fig. 2 - Experimental reduced variogram for the millimetric scale (step 0.0025 m).

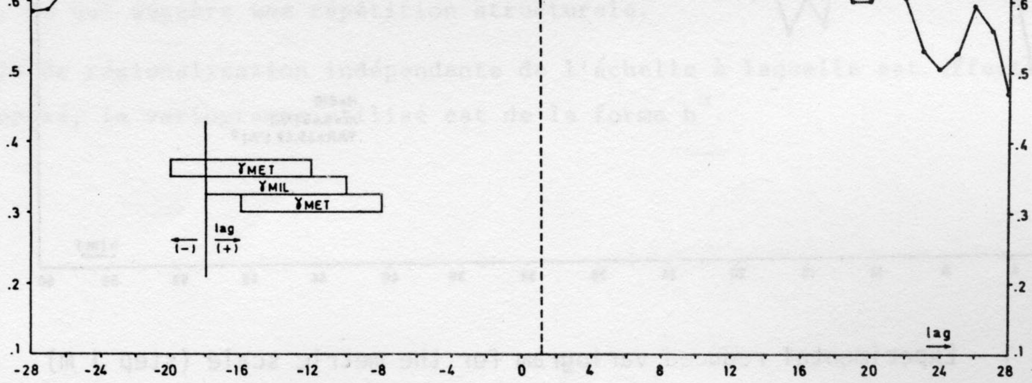

Fig. 3 - Cross correlation function relating variograms at two scales

From the simple visual comparison of the two curves (Fig. 1 and 2), a striking similarity comes up. It seems obvious that both variograms have the same general "attitude" and, apart from scale, it is felt that the two functions belong to the same "family".

In order to check this idea, values of $\gamma_{MET}(h)$ were plotted against values of $\gamma_{MIL}(h)$ for each step, and a rather good linear correlation coefficient was found (ρ = . 94). Also, the cross correlation function relating $\gamma_{MET}(h)$ to $\gamma_{MIL}(h)$ was calculated for different lags. The graph of this function is shown in Fig. 3.

The analysis of Fig. 3 leads to the conclusion that, despite of some periodicities, a large peak is reached for small lags (+ 1). So, it was decided to proceed a comparing study of the point to point variogram for different multiples of the basic step on each scale.

In order to complete experimental evidence about the two sets of data, it is also presented in Annex B typical profiles of the variable Cu grade along drill-holes and histograms for both sets (MIL and MET).

B – MODELING EXPERIMENTAL VARIOGRAMS

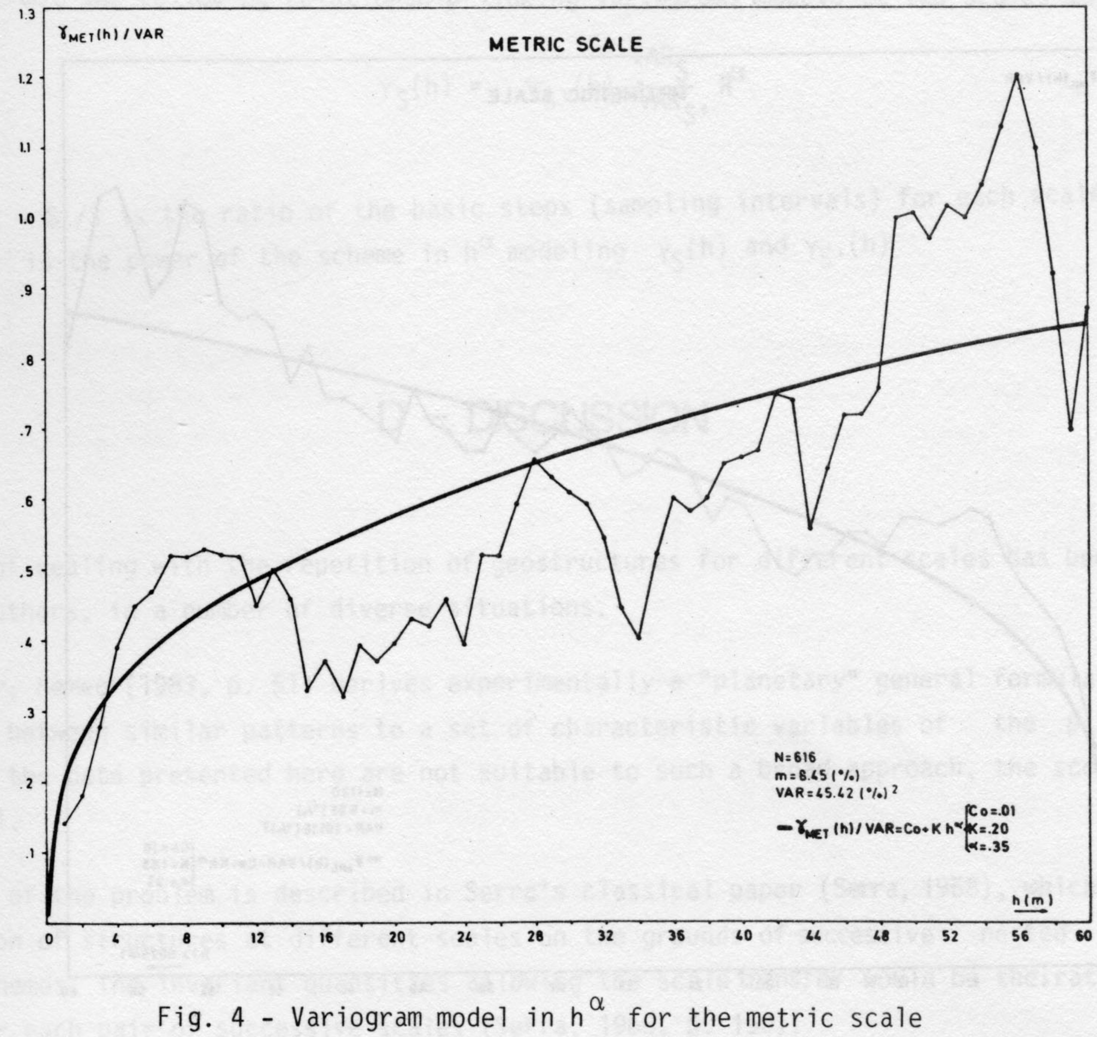

Fig. 4 - Variogram model in h^{α} for the metric scale

For the sake of simplicity, and disregarding the virtual hole-effect apparent in both variograms, the model chosen to interpolate the experimental curves is a power function scheme given by :

$$\gamma(h) = Co + K\, h^{\alpha} \qquad [1]$$

For the first set of data (MET), the fitted model is shown in figure 4. The parameters of the model are the following :

$$Co = .01$$
$$K = .20$$
$$\alpha = .35$$

For the metric scale, the variogram model is written as the following :

$$\gamma_{MET}(h)/VAR_{MET} = .01 + .20\, h^{.35} \qquad [2]$$

For the second set of data (MIL), the fitted model and the calculated parameters are presented in figure 5.

For the millimetric scale, the variogram model is written :

$$\gamma_{MIL}(h)/VAR_{MIL} = .10 + 1.63\, h^{.35} \qquad [3]$$

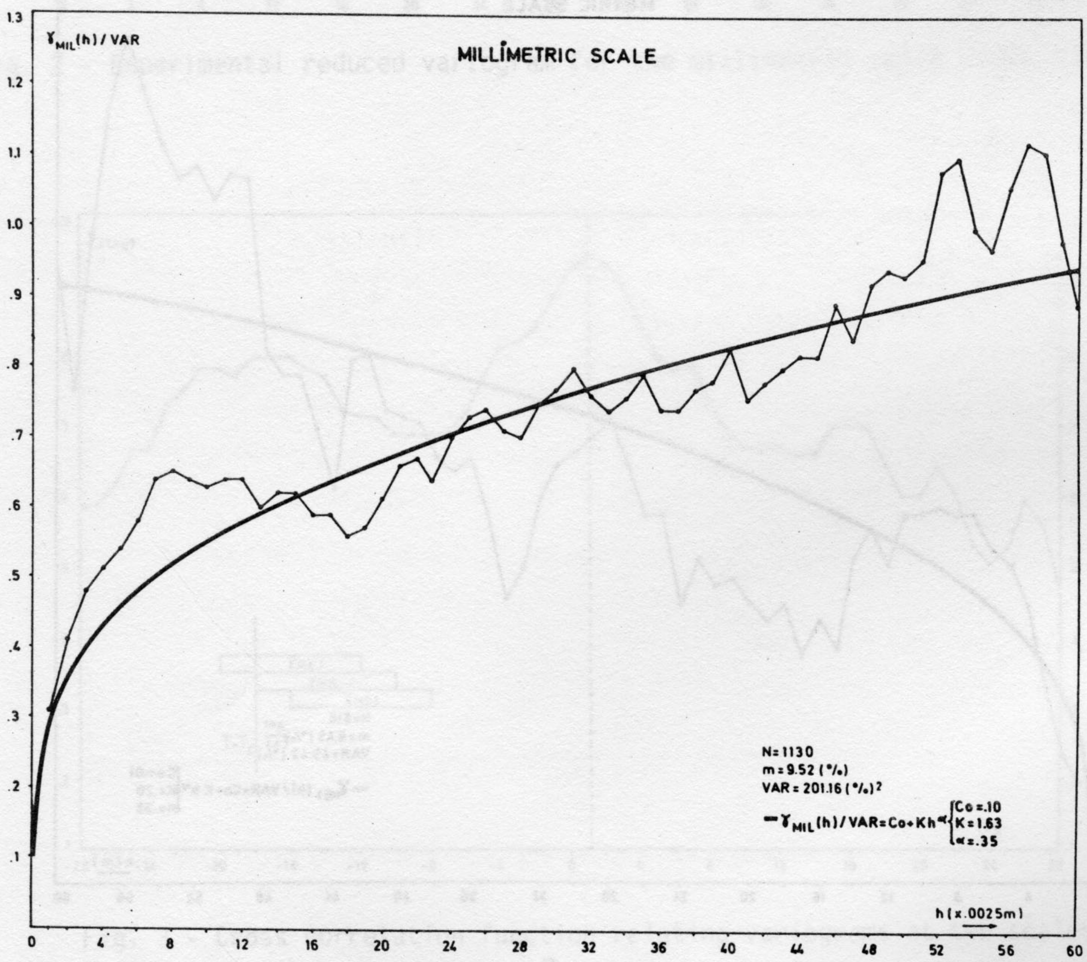

Fig. 5 - Variogram model in h^{α} for the millimetric scale

C – MATCHING VARIOGRAM MODELS AT TWO SCALES

Comparing the parameters involved in the two models previously fitted, it was noticed that the following proportional relationship holds:

$$\frac{Co_{MET}}{Co_{MIL}} \simeq \frac{K_{MET}}{K_{MIL}} \simeq \left(\frac{S_{MIL}}{S_{MET}}\right)^{.35} \simeq .1228 \qquad [4]$$

where

S_{MIL} = .0025 m is the sampling interval (basic step) at the millimetric scale

S_{MET} = 1 m is the sampling interval (basic step) at the metric scale

Hence, the millimetric variogram model may be derived from the metric one through the expression:

$$\gamma_{MIL}(h)/VAR_{MIL} = \gamma_{MET}(h)/VAR_{MET} \cdot (S_{MET}/S_{MIL})^{.35} \qquad [5]$$

In general terms, the following relationship linking variograms models at two scales may be written:

$$\gamma_S(h) = \gamma_{S'}(h) \frac{VAR_S}{VAR_{S'}} R^{\alpha} \qquad [6]$$

where

$R = S'/S$ is the ratio of the basic steps (sampling intervals) for each scale

α is the power of the scheme in h^{α} modeling $\gamma_S(h)$ and $\gamma_{S'}(h)$

D – DISCUSSION

The problem of dealing with the repetition of geostructures for different scales has been treated by several authors, in a number of diverse situations.

In particular, Nemec (1983, p. 51) derives experimentally a "planetary" general formula linking the equidistance between similar patterns to a set of characteristic variables of the planet. It is obvious that the data presented here are not suitable to such a broad approach, the scope of which is too global.

Another view of the problem is described in Serra's classical paper (Serra, 1968), which provides an interpretation of structures at different scales on the grounds of successive nested transition spherical schemes. The invariant quantities allowing the scale transfer would be the ratio of ranges and sills for each pair of successive scales (Serra, 1968, p. 150).

For the case presented here, Serra's approach may be applied if the variogram in h^{α} is taken as the sum of nested spheric models, the ranges of which are in geometric progression. In Fig. 6 and 7 it can be seen that it is possible to fit nested spheric schemes with the following ranges:

$$a_{MIL_1} = .025 \text{ m} \quad a_{MIL_2} = .375 \text{ m (millimetric scale)}$$

$$a_{MET_1} = 6 \text{ m} \quad a_{MET_2} = 84 \text{ m (metric scale)}$$

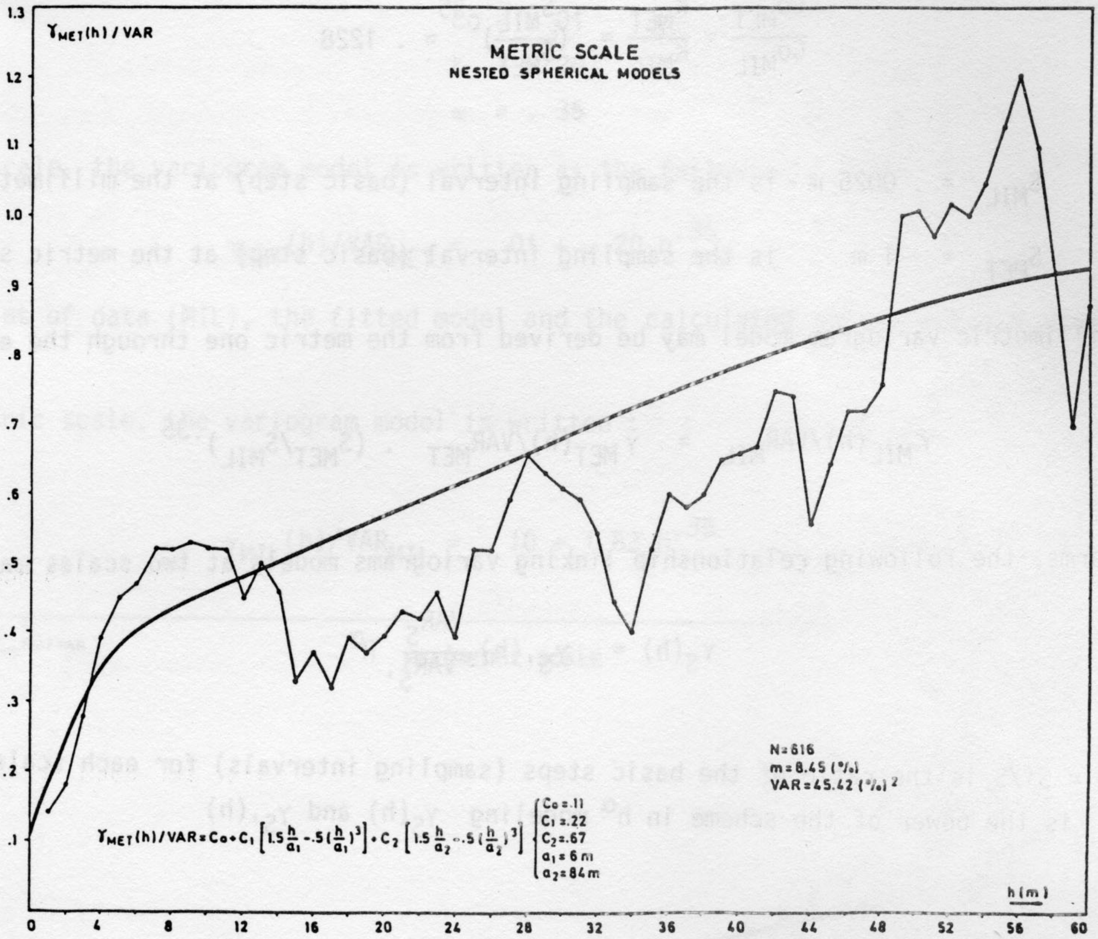

Fig. 6 - Variogram spheric nested models for the metric scale

In this particular case, a constant ratio for successive ranges (*) may be found ($a_{MIL_2}/a_{MIL_1} = a_{MET_1}/a_{MIL_2} = a_{MET_2}/a_{MET_1} \simeq 15$), although a_{MIL_2} and a_{MET_2} fall out of the experimental field available.

Moreover, this relationship does not hold for sills relative to each set of data ($C_{MIL_2}/C_{MIL_1} = 1.64$ and $C_{MET_2}/C_{MET_1} = 3.05$) and it is hard to check for different supports.

Even though a simple proportional relationship linking the parameters of the spheric models for the two scales could be found, the physical meaning of the ratio of parameters for such a relationship would not be as straightforward as the correspondent R (cf. equation [6] , where R = 400),which is the ratio of the basic steps for each scale.

(*) Assuming that ranges are insensitive to the support change

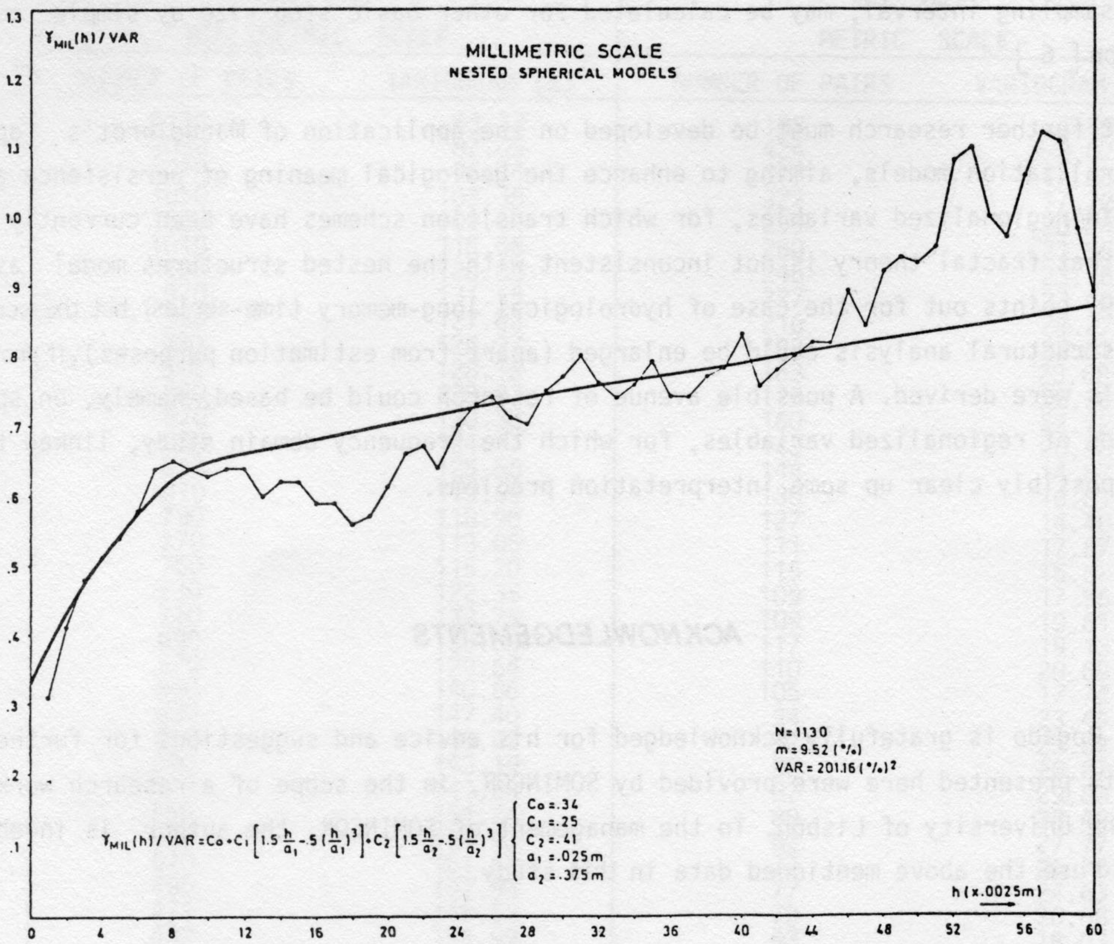

Fig. 7 - Variogram spheric nested models for the millimetric scale

So, it seems that the nested structures model, although more appealing on the geostatistical point of view (as it involves transition schemes fitting two structures apparent for each scale),presents some interpretation problems and is less parcimonious regarding the scale transfer objective.

Refering back to equation [6] , it is worth noting that the model thereby described depends only on the scale parameter R and on the power α .

The scale-free power α reveals a certain persistence in the regionalized variable spatial distribution. The physical meaning of α may be searched in the framework of Mandelbrot's theory of fractional Gaussian noise(Mandelbrot and Wallis, 1969), which is the generalization of a Wiener-Levy process (characterized by a variogram $\gamma(h) = h$). Indeed, Mandelbrot claims that the concept of "self-similarity", originated in the theory of turbulence, applies to a wide variety of natural phenomena (Mandelbrot and Wallis, 1968, p. 909). For those cases, the variogram is proportional to h^{α} , where the power α is related to the fractal dimension of the phenomenon.

Hence, in the case study presented here, the parameter α , being invariant to scale change, may be interpreted as a measure of how erratic is the profile of the copper grade along drill-holes,account ing for the strength of spatial persistence of that variable, regardless scale.

The other parameters of the variogram (vz. Co and K in equation [1]), once fitted to a certain "reference" sampling interval, may be calculated for other basic step size by simple rescaling, using equation [6]

It seems that further research must be developed on the application of Mandelbrot's approach to genetic mineralization models, aiming to enhance the geological meaning of persistence and self-similarity in regionalized variables, for which transition schemes have been currently applied. It is clear that fractal theory is not inconsistent with the nested structures model (as Hosking, 1984, p. 1899, points out for the case of hydrological long-memory time-series) but the scope of geostatistical structural analysis could be enlarged (apart from estimation purposes), if proper mineralization models were derived. A possible avenue of research could be based, namely, on spectral density functions of regionalized variables, for which the frequency domain study, linked to fractal theory, may possibly clear up some interpretation problems.

ACKNOWLEDGEMENTS

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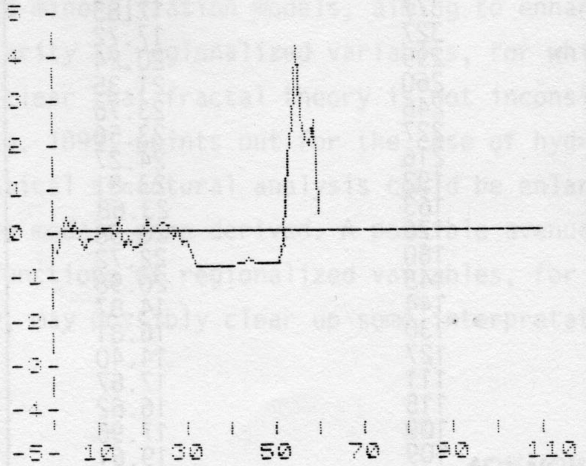
ANNEX A

TABLE OF EXPERIMENTAL VALUES FOR THE VARIOGRAM AT TWO SCALES

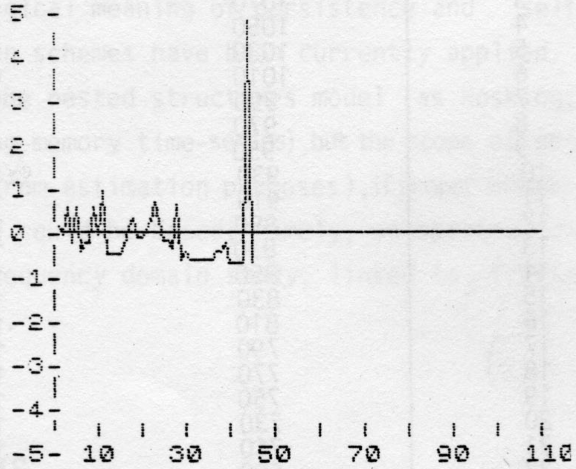
MULTIPLE OF THE BASIC STEP	SET "MIL" MILLIMETRIC SCALE		SET "MET" METRIC SCALE	
	NUMBER OF PAIRS	VARIOGRAM $(\%)^2$	NUMBER OF PAIRS	VARIOGRAM $(\%)^2$
1	1110	63.33	183	6.55
2	1090	82.05	424	8.07
3	1070	96.24	377	12.85
4	1050	102.38	327	17.72
5	1030	109.60	299	20.24
6	1010	116.55	260	21.35
7	990	128.03	246	23.76
8	970	131.55	227	23.56
9	950	127.83	216	24.27
10	930	126.37	192	23.81
11	910	128.16	163	23.68
12	890	128.93	176	20.38
13	870	120.30	160	22.72
14	850	124.17	145	20.98
15	830	125.56	144	14.87
16	810	119.59	130	16.81
17	790	118.90	127	14.40
18	770	113.05	111	17.67
19	750	115.20	115	16.62
20	730	122.35	109	17.96
21	710	132.79	109	19.61
22	690	135.60	117	19.13
23	670	129.64	110	20.69
24	650	140.86	105	17.74
25	630	147.40	94	23.44
26	610	148.82	97	23.84
27	590	143.19	92	26.87
28	570	141.29	87	29.83
29	550	151.24	88	28.49
30	530	154.93	82	27.77
31	510	161.66	77	26.78
32	490	151.93	77	24.63
33	471	148.53	70	20.08
34	452	151.99	63	18.03
35	433	159.81	58	23.80
36	414	144.23	56	27.28
37	395	149.25	53	26.25
38	376	155.67	45	27.06
39	357	157.68	42	26.90
40	338	166.26	41	29.83
41	319	153.61	37	30.34
42	300	156.25	37	33.88
43	281	161.27	33	33.73
44	262	165.15	36	25.45
45	243	164.41	32	29.07
46	225	179.62	30	32.70
47	207	169.98	27	32.85
48	190	184.90	28	34.57
49	174	189.95	27	45.22
50	159	187.47	26	45.68
51	145	192.61	24	44.24
52	133	217.24	23	46.11
53	121	221.02	20	45.40
54	109	201.67	18	47.49
55	98	194.80	16	51.31
56	89	213.78	15	55.07
57	81	225.96	14	49.85
58	73	223.22	10	41.97
59	66	196.23	7	31.65
60	50	179.01	7	39.45
BASIC STEP	0.0025 m		1 m	
NUMBER OF SAMPLES	1130		616	
AVERAGE	9.52 %		8.45 %	
VARIANCE	$201.16(\%)^2$		$45.45(\%)^2$	

A N N E X B

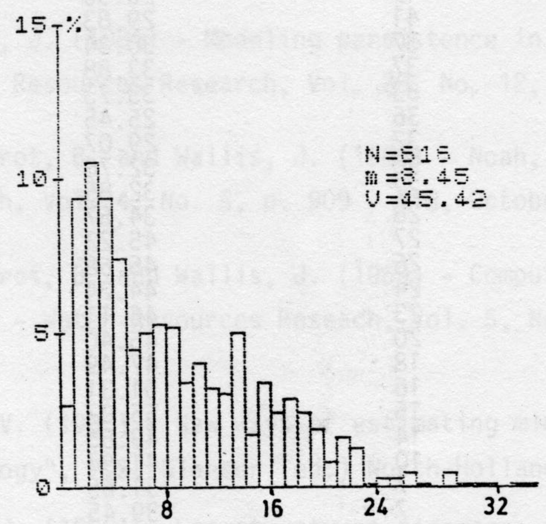
PROFILES AND HISTOGRAMS OF THE VARIABLE FOR THE TWO SETS OF DATA



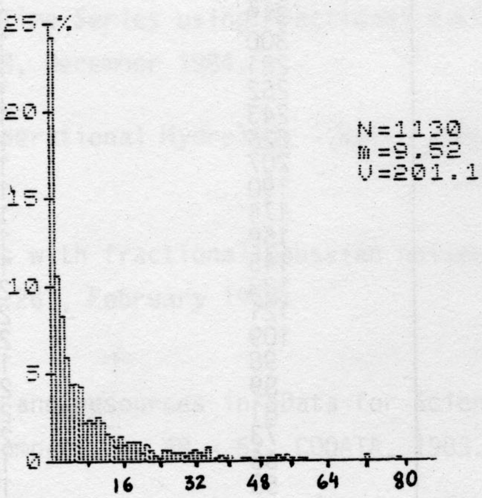
Profile of standardized variable ($z = (x - m)/\sigma$) along a typical drill-hole (set "MET")



Profile of standardized variable ($z = (x - m)/\sigma$) along a typical drill-hole (set "MIL")



Histogram of the variable for the set "MET"



Histogram of the variable for the set "MIL"