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MODEL VALIDATION IN THE ESTIMATION OF NON-STATIONARY SPATIAL PHENOMENA

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ABSTRACT

Experimental results of the application of a covariance (or variogram) model validation method for non-stationary phenomena are presented for the case of estimation of S grades in a pyrite deposit.

RESUMO

Apresentam-se os resultados experimentais da aplicação, à estimação do teor em S numa jazida de pirites, de um método de validação de um modelo de covariância (ou variograma) para fenómenos não-estacionários.

INTRODUCTION

Many spatial phenomena ocurring in Nature may be described by means of Random Functions. Characteristic variables of these phenomena (measured in some experimental points) are viewed as the realization of a Random Function whose covariance must be inferred to allow the BLUE (best linear unbiased estimation) of the variable in unknown points or surfaces. The inference of that covariance from the experimental data is not possible from one single realization of the phenomenon, when a stationarity hypothesis does not hold.

For a non-stationary phenomenon, the traditional approach to deal with estimation problems is to split the variable Z(x) (where x are the coordinates of a point in one or two dimensions) in two terms:

$$Z(x) = m(x) + Y(x)$$

Where $m(x) = E[Z(x)]$ is the **drift,** an ordinary function that accounts for the regional component of the variable and Y(x) is the **flutuation,** a stationary random functiom with zero mean and stationary covariance that models the variability of the local component of the variable.

If the covariance $C(\vec{h}) = E[Y(x+\vec{h})\ Y(x)]$ was known, the problem of estimation of the variable in an arbitrary point x_o could be solved on the grounds of the Universal Kriging theory (MATHERON, 1970), using the system:

$$\begin{cases} \lambda^{\beta} C(x_\alpha - x_\beta) = C(x_\alpha - x_o) + \mu_e f^e(x_o) & \alpha = 1, 2, \dots n \\ & [1] \\ \lambda^{\alpha} f^e(x_\alpha) = f^e(x_o) & e = 0, 1, \dots k \end{cases}$$

Where λ^α and λ^β are the weights to apply to the values of the the variable in the **n** experimental points α in order to built up the estimator $\overset{*}{Z}(x_o) = \lambda^\alpha Z(x_\alpha)$; μ_e are Lagrange parameters and f^e are monomials, depending on the degree k of the drift ($m(x) = a_e f^e(x)$). The Einstein summation convention is used in the sequel.

The kriging variance (minimum variance of $\overset{*}{Z}(x_o) - Z(x_o)$) is calculated by:

$$\sigma^2_k = C(O) + \mu_e f^e(x_o) - \lambda^\alpha C(x_\alpha - x_o) \qquad [2]$$

MODEL VALIDATION

In order to apply system [1], a covariance model must be found. In Geostatistics (MATHERON, 1970), instead of covariance, it is worth to use σ variogram function, as many phenomena display an infinite dispersion. The variogram ϑ (h) is calculated by:

$$\gamma(\vec{h}) = VAR[Y(x+h) - Y(x)] = E[Y(x+h) - Y(x)]^2$$

and system [1] holds if $C(\vec{h})$ is substituted by $-\gamma(\vec{h})$.

But the variogram function to which one has experimental acess is $\gamma_B(h) = VAR[Z(x+h) - Z(x)]$ (variogram of the original variable Z(x)) and, except in some particular cases (PEREIRA, 1983), the variogram of Y(x) can not by calculated from data, because considerable biases are introduced.

So, an a priori model for the variogram function must be set, and parameters of the model must be calculated. In this note, a method of model validation

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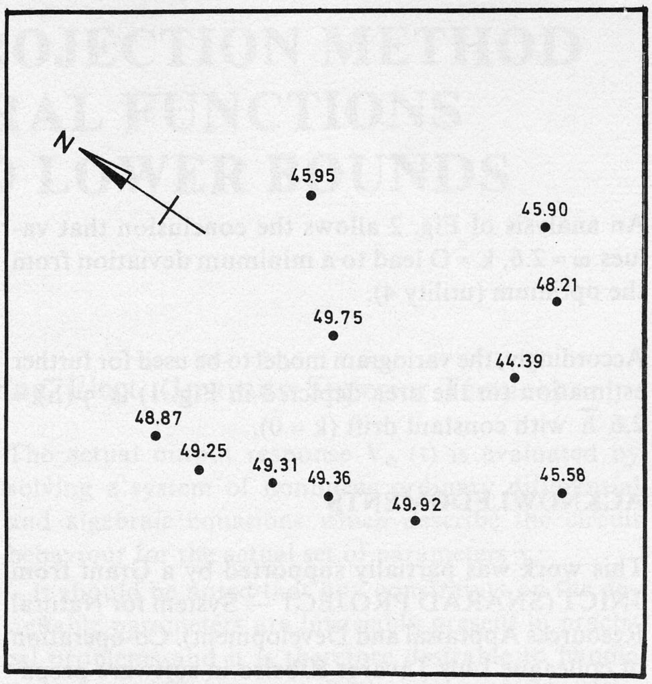
FIG. 1 — Experimental configuration of S grades (%)



is applied to experimental data of S grades in a pyrite deposit (CVRM, 1978).

The method consists in the computation of estimators $\overset{*}{Z}(x_\alpha)$ (using $\overset{*}{Z}(x_\alpha) = \lambda^\beta Z(x_\beta), \forall \beta \neq \alpha$) in all experimental points where a value of $Z(x_\alpha)$ exists. Those estimators are obtained from system [1], using the a priori model for $\gamma(\bar{h})$. The parameters of the model are choosen in a practical range whose limits are suggested by a preliminary analysis of experimental data (variograms of Z(x) and of $Z(x) - \overset{*}{m}(x)$ beeing the basis of this analysis) and by all available qualitative information about the spatial development of the phenomenon.

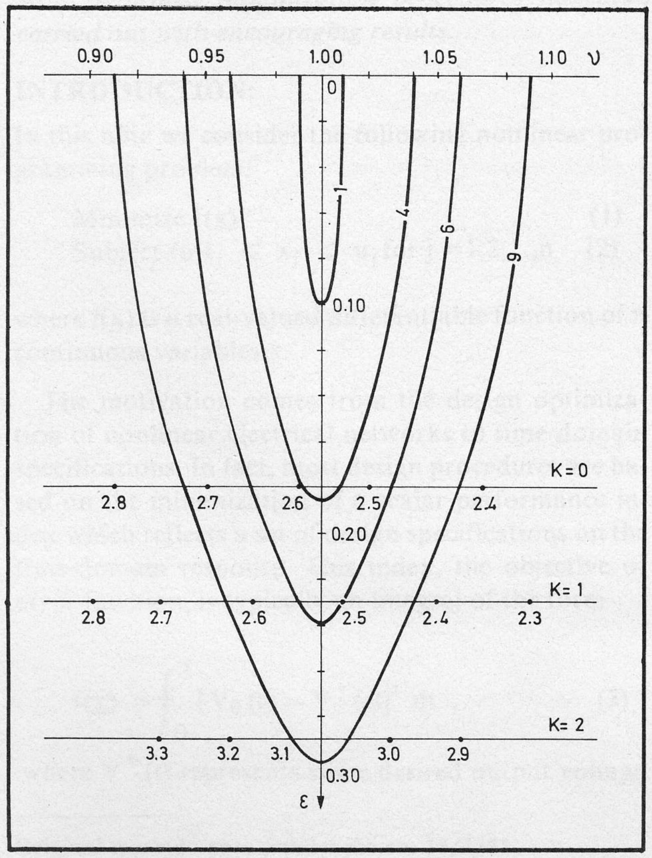
An utility surface is then built in function of two statistics:

- The Average error

$$\epsilon = \frac{1}{n} \sum_\alpha [\overset{*}{Z}(x_\alpha) - Z(x_\alpha)]$$

- The Average Relative Quadratic error

$$\nu = \frac{1}{n} \sum_\alpha [\overset{*}{Z}(x) - Z(x)]^2 \; / \; \sigma_k^2$$



The shape of indiference curves in the neighbourhood of the optimum (given by $\epsilon = 0, \nu = 1$) depends on the relative "preference" for each kind of error (the first one, ϵ, accounts for the average under or over estimation of the variable and the second one, ν, measures the goodness of fit of kriging variances calculated from the model).

The model choosen for further estimation procedures is the one for which values of statistics ϵ and ν are closest to the optimum, in the utility surface space.

EXPERIMENTAL RESULTS

The above described method was applied to an experimental configuration of 11 samples of pyrite ore assayed for S grades in a level of a pyrite deposit (Fig. 1).

The variogram model tested was $\gamma(\bar{h}) = \omega\,\bar{h}$, for ω in the range [2,3] and for k = 0, 1, 2 (degree of the drift).

In the utility surface shown in Fig. 2 (each indifference curve beeing labeled by a code 1,4,6,9 denoting a growing deviation from the optimum $\epsilon = 0$, $\nu = 1$), the experimental points for each pair of the model's parameters ω, k were plotted in function of ϵ and ν. It is worth noting that ϵ is constant for each k value (propriety of linear variogram estimation) and values of ω are figures appearing in each straight line ϵ = constant.

FIG. 2 — Plot of experimental points (ω, k) in the utility surface space

An analysis of Fig. 2 allows the conclusion that values $\omega = 2.6$, $k = O$ lead to a minimum deviation from the optimum (utility 4).

Accordingly, the variogram model to be used for further estimation (in the area depicted in Fig. 1) is $\gamma(\vec{h}) = 2.6\ \vec{h}$ with constant drift ($k = 0$).

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